

Colour-singlet clustering of partons and recombination model for hadronization of quark-gluon plasma

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$SU(N_c)$ colour-singlet restriction, along with flavour and spin symmetry, on thermal partonic ensemble is shown to recombine the partons with internal colour structure into colour-singlet multi-quark clusters which can be identified with various hadronic modes at a given temperature. This can provide a possible basis for recombination model for hadronization of quark-gluon plasma. This also leads to a natural explanation for the ratio of (anti)protons to pions and the quark number scaling of the elliptic flow coefficient in relativistic heavy-ion collisions.

PACS numbers: 12.38.Mh, 05.30.Ch, 25.75.Dw

Keywords: Quark-Gluon plasma, Colour-singlet Partition function, Recombination model, Hadronization

Introduction: By now it is generally believed that the deconfinement of strongly interacting matter has been achieved in relativistic heavy-ion collisions [1]. Remarkable confirmations for this premise come from elliptic flow of hadrons and its scaling with the number of valence quarks [2], jet quenching [3], and radiation of thermal photons [4]. However, the results from RHIC experiments also revealed some interesting facts that the nuclear suppression factor depends on the hadron species [3] and the proton to pion ratio [5] has a plateau around unity in the transverse momentum range (2 - 4) GeV/c. Recently, it has been argued in Refs. [6, 7] that the hadron production at low momenta in a dense medium takes place through the recombination of partons which explains some of the surprising results of RHIC experiments.

Generally, hadronization by recombination is a “two to one” process in a medium where the (anti)quarks are effective degrees of freedom and gluons are dynamical ones that disappear at hadronization. This simply boils down to the fact of *a correct counting of quantum states and momenta* within a dynamical theory. However, it is an extremely difficult task within the dynamical QCD and various models [6, 7, 8, 9, 10] have been formulated to describe the hadron productions in heavy-ion collisions.

Recently, under a sudden approximation the recombination is considered with a perturbative quark, *i.e.*, mini-jet and a thermal (anti)quark [7] whereas in Ref. [6] it is with thermal (anti)quarks. In Ref. [7] it is argued that an additional contribution is necessary for the transverse momentum spectra of hadron at the transition region between the thermal recombination and the individual fragmentation. Such contribution comes from the recombination of a minijet and a thermal quark. In Ref. [6], on the other hand, it is argued that for momenta below

5 GeV/c the thermal recombination dominates whereas beyond 5 GeV/c the fragmentation of independent mini-jet dominates the hadron production. Also, the competition between the recombination and fragmentation pushes the onset of fragmentation to relatively higher transverse momentum of 5 – 6 GeV/c. This indicates that the quark recombination phenomena for hadronisation of quark-gluon plasma remain an open as well an interesting problem.

In the present article, we show that the recombination phenomena arise spontaneously upon application of colour-singlet projection operator on the partition function for an assembly of quarks and antiquarks having internal colour structures. Such recombination phenomena naturally explain the baryon-to-meson ratio and the azimuthal anisotropy of hadron distributions that scales with the number of valence quarks.

Quantum statistical mechanics and colour-singlet ensemble: The statistical behaviour of a quantum gas in thermal equilibrium is usually studied through an appropriate ensemble. In general one defines a density matrix for the system as

$$\rho(\beta) = \exp(-\beta\hat{H}) \quad , \quad (1)$$

where $\beta = 1/T$ is the inverse of temperature and \hat{H} is the Hamiltonian of the physical system. The corresponding partition function for a quantum gas having a finite volume can be written as

$$\mathcal{Z} = \text{Tr} \left(\hat{\mathcal{P}} e^{-\beta\hat{H}} \right) = \sum_n \langle n | \hat{\mathcal{P}} e^{-\beta\hat{H}} | n \rangle \quad , \quad (2)$$

where $|n\rangle$ is a many-particle state in the Hilbert space \mathcal{H} and $\hat{\mathcal{P}}$ is the projection operator for any desired configuration. We propose to consider the statistical thermodynamical description of a quantum gas consisting of quarks and antiquarks, such that the underlying symmetry amounts to reordering of the partition function in terms of *the colour-singlet multi-quark modes* at a given

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temperature. We also assume here that the gluons are in thermal background and the partonic matter is mainly composed of quarks and antiquarks.

Now for a symmetry group \mathcal{G} (compact Lie group) having unitary representation $\hat{U}(g)$ in a Hilbert space \mathcal{H} , the projection operator can be written as [11, 12]

$$\hat{\mathcal{P}}_j = d_j \int_{\mathcal{G}} d\mu(g) \chi_j^*(g) \hat{U}(g) , \quad (3)$$

where d_j and χ_j are, respectively, the dimension and the character of the irreducible representation j of \mathcal{G} . $d\mu(g)$ is the normalised Haar measure in the group \mathcal{G} . The symmetry group associated with the colour-singlet configuration of the system is $SU(N_c)$, N_c is the number of colour corresponding to fundamental representation. For the $SU(N_c)$ colour-singlet configuration $d_j = 1$ and $\chi_j = 1$. Now the colour-singlet partition function for the system becomes,

$$\mathcal{Z}_S = \text{Tr} \left(\int_{SU(N_c)} d\mu(g) \hat{U}(g) \exp(-\beta \hat{H}) \right) , \quad (4)$$

where $\hat{U}(g)$ can be thought of a local link variable that links the (anti)quarks in a given state of the physical system. Now, the trace in a Fock space results in a product of the fermionic determinant over momentum modes and colour degrees of freedom, which we shall see later. Thus, the interchange of the ‘Tr’ and the integration will lead to an overall product over momentum and color states and one can write the (4) as

$$\mathcal{Z}_S = \prod \int_{SU(N_c)} d\mu(g) \text{Tr} \left(\hat{U}(g) \exp(-\beta \hat{H}) \right) . \quad (5)$$

We neglect the mutual interactions among the constituents, although they must interact in order to come to a thermal equilibrium. One can imagine a situation by first allowing them to come to a thermal equilibrium and then slowly turning off the interactions [13]. Such a simple thermodynamic description is often useful for various physical systems (*e.g.*, electrons in metal, black-body photons in a heated cavity, phonons at low temperature, neutron matter in neutron stars, etc.). The full Hamiltonian is then the sum of the Hamiltonians for each species, *i.e.*, quarks and antiquarks as $\hat{H} = \hat{H}_q + \hat{H}_{\bar{q}}$ with $\hat{H}_i = \hat{h}_i - \mu_i \hat{N}_i$, in the grand canonical ensemble with the usual meaning of μ_i and \hat{N}_i .

Now, the Hilbert space \mathcal{H} of the composite system has a structure of a tensor product of the individual Fock spaces of quarks and antiquarks as $\mathcal{H} = \mathcal{H}_q \otimes \mathcal{H}_{\bar{q}}$, where the subscripts q and \bar{q} denote the quark and antiquark, respectively. Due to this the partition function in (5) in Hilbert space \mathcal{H} decomposes into the product of two traces [12, 14, 15, 16] in their respective Fock spaces as

$$\mathcal{Z}_S = \prod \int_{SU(N_c)} d\mu(g) \text{Tr} \left(\hat{U}_q(g) e^{-\beta \hat{H}_q} \right) \text{Tr} \left(\hat{U}_{\bar{q}}(g) e^{-\beta \hat{H}_{\bar{q}}} \right) \quad (6)$$

For simplicity, we approximate the local link variable $U(g)$ to be diagonal matrix related to *only* the diagonal generators in colour space associated with maximal abelian sub-group (Cartan space) [11] of $SU(N_c)$. As for example, $SU(3)$ colour gauge group has only two parameter abelian sub-group associated with the two diagonal generators that would characterise $U(g)$, including its diagonalisation as can be seen below. Under this approximation the Haar measure corresponding to $SU(N_c)$ can now be written in the Weyl reduced [11, 12] form as

$$\int_{SU(N_c)} d\mu(g) = \frac{1}{N_c!} \left(\prod_{l=1}^{N_c} \int_{-\pi}^{\pi} \frac{d\theta_l}{2\pi} \right) \delta \left(\sum_l \theta_l \right) \times J(e^{i\theta_1} \dots e^{i\theta_{N_c}}), \quad (7)$$

where J is the Jacobian of transformation (also known as Vandermonde determinant [17]). θ_l is a class parameter characterizing the group element g such that $U(g)$ can be diagonalised. It also obeys the periodicity condition $\sum_{l=1}^{N_c} \theta_l = 0 \pmod{2\pi}$, which ensures that the group element is $SU(N_c)$. Thus, it is obvious that the product of two Fock space traces in (6) has to be a class function, which can be obtained below using the diagonalization condition in maximal abelian sub-space of $SU(N_c)$.

Now, in each Fock space there exists a basis that diagonalizes both operators as long as \hat{H}_i and $\hat{U}_i(g)$ commute. Let $|\alpha, \sigma\rangle$ be the one-particle states in such basis, where α labels the eigenvalues of \hat{H}_i (including a possible degeneracy besides the one associated with the symmetry group \mathcal{G}) and σ labels those of $\hat{U}_i(g)$. One can write the diagonalised eigenstate as

$$\langle \alpha', \sigma' | \hat{U}_i(g) \hat{A}_i | \alpha, \sigma \rangle = \delta_{\alpha\alpha'} \delta_{\sigma\sigma'} R_{i\sigma\sigma'} A_{i\alpha} . \quad (8)$$

where $A_{i\alpha}$ and $R_{i\sigma\sigma'}$ are, respectively, the eigenvalues of \hat{A}_i and $\hat{U}_i(g)$. Then following standard procedures one can obtain [12, 13]

$$\text{Tr} \left(\hat{U}_i e^{-\beta \hat{A}_i} \right) = \exp \left[\text{tr}_\alpha \text{tr}_c \ln \left(1 + R_i(g) e^{-\beta A_{i\alpha}} \right) \right]. \quad (9)$$

Note that ‘tr $_\alpha$ ’ is the trace over the momentum state α whereas ‘tr $_c$ ’ is the trace over the colour degrees of freedom in the same momentum state α . In fundamental representation $\text{tr}_c R(g^k) = \sum_{l=1}^{N_c} \exp(ik\theta_l)$ along with $R^k(g) = R(g^k)$. This can be related to the Polyakov Loop [18, 19] in Polyakov gauge as $\mathcal{L} = \text{tr}_c(L)/N_c = \text{tr}_c R(g^k)$, where L is the thermal Wilson lines defined by the temporal gluons in Euclidian time. This correspondence is due to the choice of diagonal $U(g)$, which resembles the Polyakov Loop matrix in Polyakov gauge, supplemented with the diagonalization condition in (8).

Now one can write the product of two traces in (6) as

$$\text{Tr} \left(\hat{U}_q(g) e^{-\beta \hat{H}_q} \right) \text{Tr} \left(\hat{U}_{\bar{q}}(g) e^{-\beta \hat{H}_{\bar{q}}} \right) = \exp(\Theta) , \quad (10)$$

where it is easy to show that

$$\Theta = \sum_{\alpha} \sum_l \sum_s \sum_f \left[\ln \left(1 + e^{i\theta_l} e^{-\beta(\epsilon_{q\alpha} - \mu_q)} \right) + \ln \left(1 + e^{-i\theta_l} e^{-\beta(\epsilon_{q\alpha} + \mu_q)} \right) \right]. \quad (11)$$

Here the flavour (f) and spin (s) summations are introduced, where N_f and N_s are, respectively, the number of flavour and spin degrees of freedom. The equation (11) clearly indicates the superposition of two Fock spaces [fermionic and antifermionic determinants] with particles having same momentum and internal colour structure obeying the quantum statistics. *The momentum states which do not satisfy this are automatically eliminated by the diagonalization condition in (8).* This essentially amounts to a stacking of same momentum particles. It is also evident that even if we had started by considering a free gas of quarks and antiquarks, the colour-singlet restriction related to the Polyakov Loop in Polyakov gauge links the effective degrees of freedom (anti)quarks with the surrounding thermal bath through the temporal gluons. As we will see, this allows an interesting scenario of recombinations. Now, the single-particle energy eigenvalues in a given state are same for q and \bar{q} as $\epsilon_{i\alpha} = \sqrt{p_{i\alpha}^2 + m_i^2}$. However, their occupation energies in a given state differ by their chemical potentials as $\mu_{\bar{q}} = -\mu_q = -\mu$. For convenience [15], we make a substitution $\xi = -i\beta\mu$ and $\epsilon_{q\alpha} = \epsilon_{\alpha}$ in (11) which then becomes

$$\Theta = \ln \prod_{\alpha} \prod_l \mathcal{D}, \quad \mathcal{D} = \left[e^{-\beta\epsilon_{\alpha}} (2 \cosh \beta\epsilon_{\alpha} + 2 \cos(\theta_l + \xi)) \right]^{N_f N_s}, \quad (12)$$

where the determinant \mathcal{D} is a class function. We would like to note that the class parameters in colour space get associated with the imaginary chemical potential ξ due to the choice of diagonal $U(g)$. As a consequence of which we will see below that the colour factor N_c appears with the chemical potentials indicating the number of valence (anti)quarks.

The partition function in (6) can now be written as

$$\mathcal{Z}_S = \prod_{\alpha} \int_{SU(N_c)} d\mu(g) \mathcal{D}. \quad (13)$$

The integrations on class parameters in (13) are now performed exactly by using the properties of the Jacobian and an orthonormal polynomial method [17]. The logarithm of the partition function [16] for $N_c = 3$ and two massless quarks ($N_f = 2$) in the infinite volume, V , limit reads as

$$\frac{\ln \mathcal{Z}_S}{V} = \int \frac{d^3 p}{(2\pi)^3} \ln [1 + S], \quad (14)$$

where S is the sum of the Boltzmann factors of *the colour-singlet multi-quark clusters* allowed by the symmetries

$$S = \sum_{b=0}^{2N_f} \sum_{m=\delta_{0b}}^{6N_f - N_c b} C_{mb} \exp [-(2m\epsilon + N_c b(\epsilon \mp \mu))\beta]. \quad (15)$$

In doing so we have replaced the \sum_{α} by the integration over phase space volume $d^3 x d^3 p / (2\pi)^3$. It is interesting to note that due to the colour-singlet restriction the colour factor, $N_c = 3$, and the baryon number, b , are always associated with the Boltzmann factor and thus with the partonic chemical potential indicating the excess number of quarks or antiquarks in addition to m number of quarks and antiquarks. *This happens just because the diagonal matrix, $R_{\sigma\sigma}$, in colour space or the Polyakov Loop re-adjusts the (anti)quarks of different momenta to be in a same momentum state by allowing them to exchange the momentum in the thermal bath.* As seen, in general, a given colour-singlet mode in (15) has energy $E_{mb} = (2m + N_c b)\epsilon$ having parton content $[(m + N_c b)q, m\bar{q}]$ for a hadron whereas that for anti-hadron is $[(m + N_c b)\bar{q}, mq]$. The pure mesonic modes ($b = 0$) can be identified with parton content $(mq, m\bar{q})$ having total energy $E_{m0} = 2m\epsilon$. Similarly, the pure baryonic modes ($m = 0$) have the energy $E_{0b} = N_c b\epsilon$ with parton content $[N_c b q]$ for baryon whereas that for antibaryons is $[N_c b \bar{q}]$. In the above C_{mb} is the weight factor, due to flavour and spin symmetry, appearing with each colour-singlet mesonic/baryonic/antibaryonic mode because of colour-singlet restriction. Their values for various modes are listed in Ref. [16]. For low lying mesons ($m = 1$ and $b = 0$) $C_{10} = 16$ whereas for low lying baryons and antibaryons ($m = 0$ and $b = 1$) $C_{01} = 20$, respectively. These are exact for the $SU(2)$ flavor and the $SU(2)$ spin symmetry [20] of the quark model in which nucleons and deltas are degenerate. On the other hand $m > 1$ and $b = 0$ correspond to excited mesonic modes whereas $m \geq 1$ and $b \geq 1$ are penta-quark and excited baryonic/antibaryonic modes, which are the Hagedron states [21].

Equation (14) clearly exhibits a *nontrivial* result: $SU(3)$ colour-singlet restriction, along with flavour and spin symmetry, on the quark-antiquark ensembles reorders the partition function in terms of Boltzmann factors of the colour-singlet multi-quark [mesonic/baryonic/antibaryonic] modes at any temperature. These are, however, not the bound states but can be regarded as a precursor to the confinement due to recombination [6, 7, 22]. Under a suitable confining mechanism (e.g., Polyakov Loop model [18, 19]), one can hope that these multi-quark structures could evolve into colour-singlet hadrons in the low temperature limit.

Probability and particle spectra: The probability of finding a single (anti)quark in the system in the energy interval ϵ and $\epsilon + d\epsilon$ follows from (14) as [23]

$$\mathcal{P}(\epsilon)d\epsilon = \frac{V d^3 p}{(2\pi)^3} \ln [1 + S]. \quad (16)$$

The logarithm is expanded with $S < 1$ that yields a solution $\epsilon > \zeta T$, where $\zeta \sim 1.7$ for $N_f = 2$. This provides an energy cut-off $\geq 2\zeta T$ for mesonic modes and $\geq 3\zeta T$ for baryonic modes. One can write the above as

$$\begin{aligned} \mathcal{P}(\epsilon)d\epsilon &= \sum_{b=0}^{\infty} \sum_{m=\delta_{0b}}^{\infty} C_{mb}^* \exp[-(2m\epsilon + 3b(\epsilon \mp \mu))\beta] \frac{V d^3 p}{(2\pi)^3} \\ &= \sum_{b=0}^{\infty} \sum_{m=\delta_{0b}}^{\infty} \mathcal{P}_{mb}^q(\epsilon) d\epsilon, \end{aligned} \quad (17)$$

in which all higher order terms are accumulated in C_{mb}^* , where $C_{mb}^* = C_{mb}$ for only low lying hadronic modes. In the above $\mathcal{P}_{mb}^q(\epsilon)d\epsilon$ is the probability of a single parton, with energy $\epsilon > \zeta T$, in the interval ϵ and $\epsilon + d\epsilon$ in a given mb -th mode which reads as

$$\mathcal{P}_{mb}^q(\epsilon)d\epsilon = C_{mb}^* \exp[-(2m\epsilon + 3b(\epsilon \mp \mu))\beta] \frac{V d^3 p}{(2\pi)^3}. \quad (18)$$

Now, the distribution of a parton within a given mb -th mode in the fluid in terms of the momentum of the parton ($p^\mu u_\mu = \epsilon$, u_μ is 4-velocity of the fluid) follows directly from (18) as

$$\frac{dN_{mb}^q}{d^3 x d^3 p} = \frac{C_{mb}^*}{(2\pi)^3} \exp[-(2m\epsilon + 3b(\epsilon \mp \mu))\beta]. \quad (19)$$

One can easily invert the parton momentum distribution in the above into the momentum [$P^\mu u_\mu = E_{mb} = (2m + 3b)\epsilon$] distribution of a mb -th hadronic mode as

$$\frac{dN_{mb}}{d^3 x d^3 P} = n^3 \frac{dN_{mb}^q}{d^3 x d^3 P} = \frac{C_{mb}^*}{(2\pi)^3} e^{-(E_{mb} \mp 3b\mu)\beta}, \quad (20)$$

where $n = (2m + 3b)$ and the above is the Cooper-Frye distribution [24] at the freeze-out in the rest frame of the fluid. We assume that this distribution describes the corresponding hadrons after freeze-out. The above distribution has also an important consequence: the entropy would remain conserve since the number of quarks in quark's phase space is equal to that of hadrons in hadron's phase space.

Now we can obtain the differential proton to pion ratio at central rapidity for a given transverse momentum $P_\perp \gg 3\zeta T$ as

$$\frac{dN_{01}}{dN_{10}} = \frac{dN_p}{dN_\pi} = \frac{C_{01}^*}{C_{10}^*} e^{3\mu/T} = \frac{5}{4} e^{\mu_B/T}, \quad (21)$$

where $\mu_B = 3\mu$, is the baryonic chemical potential and the above is independent of P_\perp . PHENIX data show [5] that the ratio has a plateau around unity in the P_\perp range, $2 \text{ GeV}/c \leq P_\perp \leq 4 \text{ GeV}/c$. Our estimation shows that it is in good agreement with RHIC data [5, 25]. Recall that in the recombination model by Fries et al. [6] it was found to be $(5/3) \exp(\mu_B/T)$. Similarly, antiproton to pion is $(5/4) \exp(-\mu_B/T)$ and that of antiproton to proton is $\exp(-2\mu_B/T)$, which are again in

good agreement with the RHIC data [26]. Thus, the colour-singlet projection of the thermal parton ensemble based on symmetry consideration of the underlying theory also provides a strong basis for the hadronization by recombination of quark-gluon plasma at intermediate P_\perp ($2 \text{ GeV}/c \leq P_\perp \leq 4 \text{ GeV}/c$) even though the ingredients are different from the recombination models [6, 7] with a sudden approximation.

Now the total number of a given hadron emitted by a fluid can be obtained from (19) as

$$N_{mb} = \frac{VC_{mb}^*}{(2\pi)^3} T^3 e^{-n\zeta \pm 3b\mu/T} \left[1 + n\zeta + \frac{n^2 \zeta^2}{2} \right], \quad (22)$$

which simply depends on the number of partonic degrees of freedom that forms the colour-singlet hadronic modes with $\epsilon/T > \zeta$. The ratio of the proton to pion becomes ~ 0.6 whereas it is ~ 0.3 for antiproton to pion with $\mu_B/T \sim 0.33$ in RHIC. This is consistent with the RHIC data [5].

Anisotropy and scaling: We now consider non-central collisions where the fluid has larger velocity on the x-axis (semi-minor) than on the y-axis (semi-major) leading to elliptic flow [27]. The flow coefficient is defined as

$$v_2 = \langle \cos 2\phi \rangle, \quad (23)$$

This requires that the distribution in (19) should have a nontrivial dependence on azimuthal angle ϕ . It is introduced through $\epsilon = p^\mu u_\mu = p_\perp u_0(\phi) - p_\perp u(\phi)$, where u_μ is the 4-velocity of the fluid and $u(\phi)$ can be parameterized [27] in the following form

$$u(\phi) = u + 2\alpha \cos 2\phi, \quad (24)$$

where u is ϕ -averaged of the maximum fluid 4-velocity in the ϕ direction and α specifies the magnitude of the elliptic flow, which is about 4% in a non-central Au-Au collisions at RHIC. Considering $u^\mu u_\mu = u_0^2(\phi) - u^2(\phi) = 1$ and expanding it to a first order in α , one can obtain

$$u_0(\phi) = u_0 + 2\alpha v \cos 2\phi, \quad (25)$$

where $v \equiv u/u_0$. Now, the elliptic flow coefficient for the mb -th mode in (23) is found to scale perfectly with the partonic p_\perp as

$$(v_2)_{mb} = \frac{\alpha}{T} (1 - v) (2m + 3b) p_\perp. \quad (26)$$

We note that this scaling is strictly valid in the recombination region ($2 \text{ GeV}/c \leq P_\perp \leq 4 \text{ GeV}/c$), where the mass dependence of the hadrons is irrelevant. However, for $P_\perp < 1.5 \text{ GeV}/c$ the scaling deviates [2] due to the mass ordering of the hadrons. To demonstrate this fact one needs a dynamical mass generating term of the quarks, which is, however, beyond scope of this calculation.

Summary: We propose that the recombination phenomena occurs readily when one constructs a colour-singlet

partition function from a thermal ensemble of quarks and antiquarks having internal symmetries, *viz.*, colour, spin and flavour. This colour-singlet projection is shown to be related to the Polyakov Loop in the Polyakov gauge, which plays the important role for recombination. We also show that such recombination of thermal partons naturally describes some of the puzzling results in RHIC experiments, which, in turn, strongly supports that the recombination of thermal (anti)quarks could be one of the dominant mechanisms of hadronization in a dense

medium in the intermediate transverse momentum of hadrons ($2 \text{ GeV}/c \leq P_{\perp} \leq 4 \text{ GeV}/c$). Nevertheless, to test the various quark recombination models for hadronisation from dynamical QCD still remain an open question. We, however, tried to address this question based on the symmetry consideration of the theory, which may be useful for the eventual solution of this important problem.

Acknowledgment: Valuable discussions with B. Müller, D. K. Srivastava, S. Bass and R. Ray are acknowledged.

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